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ON DISTRIBUTED PORT-HAMILTONIAN PROCESS SYSTEMS

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Abstract: In this paper we use the term distributed port-Hamiltonian Process Systems (DPHPS) to refer to the result of merging the theory of distributed Port-Hamiltonian systems (DPHS) with the theory of process systems (PS). Such concept is useful for combining the systematic interconnection of PHS with the thermodynamic-passivity properties of PS. Furthermore the systematic methods of control developed for PHS are then expected to be applicable for process systems. *Copyright 2004 ©IFAC*

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1. INTRODUCTION

There is a certain confluence of the theory of Hamiltonian systems and dissipativity theory (Willems, 1972) and process control theory (Hangos *et al.*, 2001), associated to the possibility of providing a fairly uniform framework for the representation, modelling, interconnection and control of the so-called *process systems* (Alonso and Ydstie, 1996). This includes both lumped and distributed parameter system representations, which as a class of models may comprise a rather wide variety of chemical unit operations. In the case of distributed port-Hamiltonian systems (DPHS) there are several recent important progresses on this theoretical framework especially for modelling (van der Schaft and Maschke, 2001), (van der Schaft and Maschke, 2002) and simulation (Golo *et al.*, 2003; Clemente-Gallardo *et al.*, 2002).

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A process system (PS) is a system based on conservation laws which supports the laws of thermodynamics (Ydstie and Alonso, 1997). The theory of PS has been formalized in a series of papers from the chemical engineering and process control community, see (Ydstie, 2002) and references therein. Several concepts of PS theory are constructed on the theory of passive and dissipative systems. In this manner while in dissipativity theory one refers to *storage functions* as generalizations of stored energy or entropy (Willems, 1972), in PS it is defined the term *inventory* to include the storage of extensive thermodynamic properties in general *e.g.* mass storage or mass component storage. An *action* is defined as any interchange of mass and energy with the environment.
Since entropy cannot be used as a storage function, several works have been devoted to find an adequate storage function which reconciles the classical theory of thermodynamics with dissipativity theory. In (Alonso and Ydstie, 2001) instead of entropy, a storage function (inventory) composed of internal energy

u and entropy s in a form similar to the Helmholtz free energy or *thermodynamic availability* is proposed since it is bounded and shown to satisfy the dissipation inequality, preluding thus passivity properties. Furthermore such properties have been used with (passivity-based) control purposes, see (Alonso and Ydstie, 2001) and references therein. Regarding the use of the Hamiltonian framework for PS, in (Hangos *et al.*, 2001) a framework for the modelling of simple Hamiltonian systems is presented. In (Ydstie, 2002) further steps have been performed towards a variational principle of non-equilibrium thermodynamics for PS.

In this paper we contribute in merging several passivity concepts reported in the references of process systems with those of port-Hamiltonian systems. The resulting methodology may show to be advantageous by providing a systematic framework for modelling, simulation and control of PS.

The paper is organized as follows: after presenting basic concepts of process systems and dissipativity theory in Section 2, elemental theory for distributed port-Hamiltonian systems is presented in Section 3 and exemplified with particular cases of Euler's fluid equations.

Notation. Brackets distinguish functionals $F[f(t)]$ from functions $f(t)$. Material, partial and variational derivatives are denoted by $\frac{DM}{Dt} = D_t M$, $\frac{\partial F}{\partial x} = \partial_x F$ and $\frac{\delta \mathcal{H}}{\delta x} = \delta_x \mathcal{H}$ respectively. The exterior differential operator $d : \bigwedge^p(\mathcal{M}) \to \bigwedge^{p+1}(\mathcal{M})$ is locally expressed by $d = \partial/\partial x$. Denote by $\forall(t)$ a closed region $\Omega \in \mathbb{R}^3$ called *control volume* and denote $A(t) = \partial\forall(t)$ the *control surface* enclosing $\forall(t)$. A *pathline* is the trajectory of a fluid particle when moving in a region Ω, *i.e.* $x(t) = \phi(x_0, t)$ such that $x_0 = \phi(x_0, 0)$. By $(\mathbf{v} \cdot \mathbf{n})$ denote the inner product of the velocity vector $\mathbf{v}$ and the unitary vector $\mathbf{n}$, outward normal to $\partial\forall(t)$. Denote $\psi(z)$ the specific potential energy, typically $\psi(z) = gz$, with g the acceleration of gravity and z the altitude. Classical thermodynamic properties are classified as extensive and intensive. Loosely speaking, while the former are properties proportional to the quantity of material in a system, *e.g.* volume $\forall$, total energy (Hamiltonian) $\mathcal{H}$, total mass m, momentum M, entropy S *etc.*, the latter are independent of the quantity of material and not additive for the various phases of the system, *e.g.* pressure P, temperature T, and densities (properties per unit volume): mass density ρ, *heat flow density* q, *etc*. Specific quantities (properties per unit mass) are expressed in lower-case, specific entropy s, specific internal energy u, specific Hamiltonian h, *etc*.

2. DISSIPATIVITY THEORY AND PROCESS SYSTEMS

In this section, dissipativity theory for process systems is reviewed with the purpose of fitting this theoretical framework to the theory of distributed port-Hamiltonian systems.

In contrast with standard dissipativity theory, where the supply rate is defined in terms of input-output variables, for a distributed process system represented by Σ, the supply rate is defined in terms of intensive variables as follows.

Definition 2.1. Let $r(x(t))$ be a scalar field called *supply rate density* and $\forall(t)$ be a closed volume, then the functional

$$S[x(t)] = \int_{\forall(t)} r(x(t))\, d\forall, \tag{1}$$

is called a *storage functional*.

The interpretation of extensive and intensive properties as storage functions and supply rates in the context of dissipativity theory was introduced by (Alonso and Ydstie, 1996). While $S[x(t)]$ represents extensive properties, its density $r(x(t))$ represents intensive properties. For instance the total mass, momentum and Hamiltonian are related with their respective supply rate densities as in Table 1.

An interesting consequence of the classic result known as the Transport Theorem (for instance, refer to (Shivamoggi, 1998)) is the definition of a dissipation inequality as is shown in the sequel. Essentially, the Transport Theorem asserts that, under certain conditions, in a region $\forall_0$ the rate of change of the storage function $S[x]$ over a material volume $\forall$ equals the rate of change of $S[x]$ over the fixed volume (which coincides momentarily with $\forall$) plus the flux of $S[x]$ leaving $\partial\forall$ otherwise expressed

$$\frac{D}{Dt} S[x(t)] = \int_{\forall(t)} \frac{\partial r}{\partial t} d\forall + \int_{A(t)} r\, \mathbf{v} \cdot \mathbf{n}\, dA(t) \tag{2}$$

where $\forall(t) = \phi(\forall_0, t)$. The Transport Theorem can be considered as a differential representation of the dissipation inequality for conservative systems. Furthermore since it can be verified that assuming that $S[x(t)]$ is conserved

$$S[x(t)] = \int \left\{ \int_{\forall} \frac{\partial r}{\partial t} d\forall \right\} dt, \tag{3}$$

is a storage function and time integration on both sides of Eq. (2) provides an equality equivalent to Willems' dissipation inequality (Willems, 1972) for conservative systems where the *supply rate functional*

$$R[r, \mathbf{v}, A] = -\int_A r\, \mathbf{v} \cdot \mathbf{n}\, dA, \tag{4}$$

plays the role of supply rate. This functional R is directly associated to the surfaces at the boundaries where the inputs and outputs interact with the system.

Definition 2.2. (Distributed dissipation inequality). A distributed parameter system Σ in a region $\forall$ of $\mathbb{R}^3$ is

said to be dissipative with respect to the supply rate functional R, defined by Eq. (4) if there exists a functional $S: \mathcal{X} \to \mathbb{R}^+$, called storage functional defined by Eq. (3), such that for all $x_0 \in \mathcal{X}$, all $t_1 \geq t_0$ and all possibly non-connected input-output bounding surfaces the inequality

$$S[x(t_1)] - S[x(t_0)] \leq \int_{t_0}^{t_1} R[r, \mathbf{v}, A]dt, \qquad (5)$$

is preserved.

This inequality plays an important role in the definition of dissipativity in process systems. In this paper we have preferred to keep the structure of Willems' dissipativity inequality, which differs from the approach taken in the references of process systems.

Remark 2.1. Following the notation of (Alonso and Ydstie, 1996), define as action $R = \phi(t, \tau)$ and the production as $p = S(x)$. Then the process $\pi = v(t + \tau) - v(t) = p + \phi$ defines a PS. Additional definitions like inventories, conservative and Clausius systems follow from this (Alonso and Ydstie, 1996).

Definition 2.3. Consider the distributed parameters system Σ in a region $\forall$ of Ω. Define the *dissipation rate functional*

$$D[d, \mathbf{v}, A] = \int_A d\ \mathbf{v} \cdot \mathbf{n}\ dA + \int_{\forall} d_d\ d\forall, \qquad (6)$$

where the scalar function d is called *dissipation rate density* and d_d *distributed dissipation rate density*, associated to the storage functional $S: \mathcal{X} \to \mathbb{R}^+$ as defined by Eq. (3), such that for all $x_0 \in \mathcal{X}$, all $t_1 \geq t_0$ and all possibly non-connected input-output boundary surfaces, the following relation

$$S[x(t_1)] - S[x(t_0)] = \int_{t_0}^{t_1} (R - D)dt, \qquad (7)$$

holds.

Definition 2.4. With regard to the distributed system Σ with supply rate functional R, assuming that the following objects exist [2], the *available storage functional*, $S_a : \mathcal{X} \to \mathbb{R}^+$, is defined as

$$S_a[x] = - \inf_{\substack{u_b(\cdot) \in \mathcal{U} \\ x_0 = x,\ T \geq 0}} \int_0^T R[r(x), \mathbf{v}(t), A(t)]dt,$$

and the *required supply functional*, $S_r : \mathcal{X} \to \mathbb{R}^+$ is defined as

[2] As known, variational principles for infinite dimensional systems suffer from several technical difficulties merely on the question of existence.

Table 1. A few functionals

	$r(x(t))$	$-R[r, \mathbf{v}, A]$	$S(x(t))$
Mass	ρ	$\int_A \rho\ \mathbf{v} \cdot \mathbf{n}\ dA$	$m = \int_{\forall} \rho\ d\forall$
Momentum	ρv	$\int_A \rho v\ \mathbf{v} \cdot \mathbf{n}\ dA$	$M = \int_{\forall} \rho v\ d\forall$
Hamiltonian	ρh	$\int_A \rho h\ \mathbf{v} \cdot \mathbf{n}\ dA$	$\mathcal{H} = \int_{\forall} \rho h\ d\forall$

$$S_r[x] = \inf_{\substack{u_b(\cdot) \in \mathcal{U} \\ x_0 = x,\ T \geq 0}} \int_{-T}^{0} R[r(x), \mathbf{v}(t), A(t)]dt,$$

where $u_b(\cdot) \in \mathcal{U}$ denotes a boundary input, $R[r, \mathbf{v}, A]$, is the supply rate functional and $r(x(t))$, is the supply-rate density.

Example 2.1. Consider a process system consisting of a fluid enclosed in a volume $\forall$ whose boundary $\partial\forall(t)$ allows for energy exchange with its surroundings. As known, if $C_{\forall}$ denotes the heat coefficient and $\Phi = \tau\partial_x v$ the dissipation function, the dynamical equations (further notation is explained at the end of Section 1)

$$D_t\rho = -\rho\partial_x v, \qquad (8)$$
$$\rho D_t v = -\partial_x P - \rho\partial_x \psi, \qquad (9)$$
$$\rho D_t s = \Phi - \partial_x q, \qquad (10)$$
$$\rho D_t u = -\Phi - P\partial_x v - \partial_x q, \qquad (11)$$
$$\rho C_{\forall} D_t T = -\Phi - \partial_x q, \qquad (12)$$

can be deduced with the help of among others storage functionals as defined in Table 1 and the Transport Theorem (2). Such dynamic equations satisfy (5) or (7) by construction. Furthermore, two possible energy storage functionals for the system as a whole may be conceived for this system. If the energy dissipated or converted into entropy is not stored in the system, one natural storage functional is

$$\mathcal{H}[\rho, v, u, T] = \int_{\forall} \left\{ \frac{1}{2}\rho v \cdot v + \rho u(\rho, T) + \rho\psi(z) \right\} d\forall \qquad (13)$$

Otherwise, if the energy stored as entropy is included, the storage functional is

$$\mathcal{H}[\rho, v, s, T] = \int_{\forall} \left\{ \frac{1}{2}\rho v \cdot v + \rho u(\rho, T) + \rho\psi(z) \right\} d\forall + \int_{\forall} \rho s T\ d\forall. \qquad (14)$$

The last integral includes other forms of energy larger than that given by the entropy (irrecoverable energy) since its derivative with respect to time yields $\int_{\forall} \{(\partial_t\rho)sT + \rho(\partial_t s)T + \rho s(\partial_t T)\} d\forall = \dot{S}T + \int_{\forall} \rho s(\partial_t T)\ d\forall$. The first term corresponds to a power pair associated to the storage of entropy and the last integral accounts for variations of T.

In the case of a conservative system, a dissipation

equality (7) holds with a supply rate functional given as in Table 1 which can be written as

$$R[\rho h, \mathbf{v}, A] = -\int_{\forall} \nabla_x(\rho h \mathbf{v}) d\forall. \tag{15}$$

This relation provides the input-output interaction of the system and is directly related with the concept of ports in PHS, discussed in the following section.

3. DISTRIBUTED PORT-HAMILTONIAN PROCESS SYSTEMS

In (van der Schaft and Maschke, 2001; van der Schaft and Maschke, 2002) the Stokes-Dirac structures are shown to be useful in providing an adequate geometric framework to represent DPHS on a bounded spatial domain. Based on such Stokes-Dirac structures, in (Clemente-Gallardo *et al.*, 2002), an extension of these descriptions is presented for the Euler equations in the case of a one-dimensional manifold with boundary where (ρ, v, s) are the dynamic variables assuming that there is no interchange of flow of heat at the boundaries. In the following result an improvement on the result presented in (Clemente-Gallardo *et al.*, 2002) is shown. Consider again the PS in Example 2.1 where

$$\partial_t \mathcal{H}[\rho, v, s, T] = \int_{\forall} \left\{ \partial_t^T \rho \; \delta_\rho \mathcal{H} + \partial_t^T v \; \delta_v \mathcal{H} + \partial_t^T s \; \delta_s \mathcal{H} + \partial_t^T T \; \delta_T \mathcal{H} \right\} d\forall. \tag{16}$$

It is possible to define two spaces of forms. Given a 1-dimensional Riemannian manifold $\mathcal{M}$ with metric g and $0-$dimensional boundary $\partial\mathcal{M}$, consider two spaces $\mathcal{F}^a$ and $\mathcal{E}^a$

$$\begin{aligned}
\mathcal{F}^a &= \mathcal{F}_\rho \times \mathcal{F}_v \times \mathcal{F}_s \times \mathcal{F}_T \times \mathcal{F}_b \times \mathcal{F}_d \\
&= \bigwedge^1(\mathcal{M}) \times \bigwedge^1(\mathcal{M}) \times \bigwedge^0(\mathcal{M}) \times \bigwedge^0(\mathcal{M}) \\
&\quad \times \bigwedge^0(\partial\mathcal{M}) \times \bigwedge^1(\mathcal{N}), \\
\mathcal{E}^a &= \mathcal{E}_\rho \times \mathcal{E}_v \times \mathcal{E}_s \times \mathcal{E}_T \times \mathcal{E}_b \times \mathcal{E}_d \\
&= \bigwedge^0(\mathcal{M}) \times \bigwedge^0(\mathcal{M}) \times \bigwedge^1(\mathcal{M}) \times \bigwedge^1(\mathcal{M}) \\
&\quad \times \bigwedge^0(\partial\mathcal{M}) \times \bigwedge^0(\mathcal{N}).
\end{aligned}$$

where the subindexes b and d stand for boundary variables and distributed variables. Such distributed variables are associated to the space $\mathcal{N} = \mathcal{M} \backslash \partial\mathcal{M}$. Throughout the section we denote

$$\begin{cases} f^i &= (f^i_\rho, f^i_v, f^i_s, f^i_T, f^i_b, f^i_d) \in \mathcal{F}^a \\ e^i &= (e^i_\rho, e^i_v, e^i_s, e^i_T, e^i_b, e^i_d) \in \mathcal{E}^a \end{cases}$$

along with a product $\langle\langle (f^1, e^1), (f^2, e^2) \rangle\rangle$ in the space $\mathcal{F}^a \times \mathcal{E}^a$ defined as

$$\begin{aligned}
&\int_{\mathcal{M}} \{ e^1_\rho \wedge f^2_\rho + e^1_v \wedge f^2_v + e^1_s \wedge f^2_s + e^1_T \wedge f^2_T \\
&\quad + e^2_\rho \wedge f^1_\rho + e^2_v \wedge f^1_v + e^2_s \wedge f^1_s + e^2_T \wedge f^1_T \} \\
&+ \int_{\partial\mathcal{M}} (e^1_b \wedge f^2_b + e^2_b \wedge f^1_b) + \int_{\mathcal{N}} (e^1_d \wedge f^2_d + e^2_d \wedge f^1_d).
\end{aligned} \tag{17}$$

Theorem 3.1. The subspace D^a defined by

$$\begin{bmatrix} f_\rho \\ f_v \\ f_s \\ f_T \\ e_d \end{bmatrix} = \begin{bmatrix} 0 & -d & 0 & 0 & 0 \\ -d & 0 & \xi & \zeta & 0 \\ 0 & -\xi & 0 & 0 & -\vartheta \\ 0 & -\zeta & 0 & 0 & -\varphi \\ 0 & 0 & \vartheta & \varphi & 0 \end{bmatrix} \begin{bmatrix} e_\rho \\ e_v \\ e_s \\ e_T \\ f_d \end{bmatrix}, \tag{18}$$

$$\begin{bmatrix} f_b \\ e_b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_\rho|_{\partial\mathcal{M}} \\ e_v|_{\partial\mathcal{M}} \end{bmatrix}, \tag{19}$$

defines a Dirac structure on $\mathcal{F}^a \times \mathcal{E}^a$, where $\xi = (*ds)(*\rho)^{-1}$, $\zeta = (*dT)(*\rho)^{-1}$, $\vartheta = 1/(\rho T)$, $\varphi = 1/(\rho C_{\mathbf{v}})$ and $*$ is the Hodge operator associated to the Riemannian structure in $\mathcal{M}$.

Proof. The proof is a variation of the one presented on (Clemente-Gallardo *et al.*, 2002; van der Schaft and Maschke, 2001). Denote with tilde accent variables evaluated at the boundary, $\tilde{x} = x|_{\partial\mathcal{M}}$. Given a subspace $D^a \subset \mathcal{F}^a \times \mathcal{E}^a$, the space $D^{a\perp}$ is defined by

$$\begin{aligned}
\{(f, e) \in \mathcal{F}^a \times \mathcal{E}^a \mid \langle\langle (f, e), (f_{D^a}, e_{D^a}) \rangle\rangle = 0 \\
\forall (f_{D^a}, e_{D^a}) \in D^a \}.
\end{aligned} \tag{20}$$

The proof that $D^{a\perp} = D^a$ is made in two steps.
(a) $D^a \subset D^{a\perp}$: Given the points $(f, e) \in D^a$, we have to prove that $\langle\langle (f, e), (f_{D^a}, e_{D^a}) \rangle\rangle = 0$. Let $(f^1, e^1) \in D^a$ and take any $(f^2, e^2) \in D^a$. The resulting product $\langle\langle (f^1, e^1), (f^2, e^2) \rangle\rangle$ is given by Eq. (17). Since the points belong to D^a, the expressions of the flows can be replaced by the corresponding expressions in terms of the efforts, yielding

$$\begin{aligned}
&\int_{\mathcal{M}} (-e^1_\rho \wedge de^2_v - e^1_v \wedge de^2_\rho + e^1_v \wedge \xi e^2_s + e^1_v \wedge \zeta e^2_T \\
&- e^1_s \wedge \xi e^2_v - e^1_s \wedge \vartheta f^2_d - e^1_T \wedge \zeta e^2_v - e^1_T \wedge \varphi f^2_d - e^2_\rho \wedge de^1_v \\
&- e^2_v \wedge de^1_\rho + e^2_v \wedge \xi e^1_s + e^2_v \wedge \zeta e^1_T - e^2_s \wedge \xi e^1_v - e^2_s \wedge \vartheta f^1_d \\
&- e^2_T \wedge \zeta e^1_v - e^2_T \wedge \varphi f^1_d) + \int_{\partial\mathcal{M}} (\tilde{e}^1_v \wedge \tilde{e}^2_\rho + \tilde{e}^2_v \wedge \tilde{e}^1_\rho) + \\
&\int_{\mathcal{N}} (\vartheta de^1_s \wedge f^2_d + \varphi de^1_T \wedge f^2_d + \vartheta de^2_s \wedge f^1_d + \varphi de^2_T \wedge f^1_d).
\end{aligned}$$

Using Stokes' theorem, (van der Schaft and Maschke, 2001) it can be seen that

$$\begin{aligned}
\int_{\partial\mathcal{M}} (e^1_\rho \wedge e^2_v + e^1_v \wedge e^2_\rho) = \int_{\mathcal{M}} (e^1_\rho \wedge de^2_v \\
+ e^1_v \wedge de^2_\rho + e^2_\rho \wedge de^1_v + e^2_v \wedge de^1_\rho).
\end{aligned}$$

Furthermore, those terms associated to the entropy are cancelled (Clemente-Gallardo *et al.*, 2002) since $e_v \in C^\infty(M)$ and $e_s \in \bigwedge^1(\mathcal{M})$ it can be asserted that the following relations are true $e^1_s \wedge \xi e^2_v = e^2_v \wedge \xi e^1_s$ and

$e_v^1 \wedge \xi e_s^2 = e_s^2 \wedge \xi e_v^1$. In our case, since $e_T \in \bigwedge^1(\mathcal{M})$ then the additional relations $e_T^1 \wedge \zeta e_v^2 = e_v^2 \wedge \zeta e_T^1$ and $e_v^1 \wedge \zeta e_T^2 = e_T^2 \wedge \zeta e_v^1$ cancel out all the remaining terms. With the same approach and performing the operations in $\mathcal{N} \subset \mathcal{M}$, those terms involving $f_d \in \bigwedge^1(\mathcal{N})$ with e_s and e_T are cancelled too.

(b) $D^{a\perp} \subset D^a$: Given two points $(f^2, e^2) \in D^{a\perp}$, $(f^1, e^1) \in D^a$, we have to show that $(f^2, e^2) \in D^a$. Consider the product $\langle\langle (f^1, e^1), (f^2, e^2) \rangle\rangle = 0$ as given by Eq. (17). The flows $(f_\rho^1, f_v^1, f_s^1, f_T^1, f_b^1, f_d^1)$ can be replaced by their corresponding expressions in the structure in Theorem 3.1 in terms of the efforts, but in contrast with the first part of the proof, those of the point (f^2, e^2) cannot be replaced. It yields

$$\begin{aligned}
&\int_{\mathcal{M}} \{e_\rho^1 \wedge f_\rho^2 + e_v^1 \wedge f_v^2 + e_s^1 \wedge f_s^2 + e_T^1 \wedge f_T^2 \\
&- e_\rho^2 \wedge de_v^1 - e_v^2 \wedge de_\rho^1 + e_v^2 \wedge \xi e_s^1 + e_v^2 \wedge \zeta e_T^1 \\
&- e_s^2 \wedge \xi e_v^1 - e_s^2 \wedge \vartheta f_d^1 - e_T^2 \wedge \zeta e_v^1 - e_T^2 \wedge \varphi f_d^1\} + \\
&\int_{\partial\mathcal{M}} (\tilde{e}_v^1 \wedge f_b^2 + e_b^2 \wedge \tilde{e}_\rho^1) + \int_{\mathcal{N}} [(\vartheta e_s^1 + \varphi e_T^1) \wedge f_d^2 + e_d^2 \wedge f_d^1].
\end{aligned}$$

Since the elements of each factor commute (each consist of a 1-form and a function), we may obtain the coefficients of the equivalent monomials by comparison. For instance by comparing $e_\rho^1 \wedge f_\rho^2$ with $e_\rho^2 \wedge (-de_v^1)$ it can be inferred that $f_\rho^2 = -de_v^2$. Doing the same with the efforts e_v^1, e_s^1, e_T^1, e_b^1, f_b^1 and f_d^1 we obtain the following

$$\begin{cases} f_\rho^2 = -de_v^2 \\ f_v^2 = -de_\rho^2 + \xi e_s^2 + \zeta e_t^2 \\ f_s^2 = -\xi e_v^2 - \vartheta f_d^2 \\ f_T^2 = -\zeta e_v^2 - \varphi f_d^2 \end{cases} \quad \begin{cases} f_b^2 = e_\rho^2|_{\partial\mathcal{M}} \\ e_b^2 = e_v^2|_{\partial\mathcal{M}} \\ e_d^2 - \vartheta e_s^2 + \varphi e_T^2 \end{cases}$$

and the proof is complete. ■

In the rest of the section a DPHS representing the 1-D version of the Euler equations ($\tau = 0$) from Example 2.1 is presented. As will be explained further on, such fluid equations are approximations of the complete set of Euler equations for some of the equations.

Proposition 3.1. Assume that a fluid has a storage of energy given by (14), then the 1-D fluid equations with external (distributed) heat flow (8), (9) and (12) with modified momentum equation given by

$$\rho D_t v = -\partial_\rho P \partial_x \rho - \rho \partial_x \psi \tag{21}$$

can be expressed in a port-Hamiltonian form by the Stokes-Dirac Structure

$$\begin{bmatrix} \partial_t \rho \\ \partial_t v \\ \partial_t s \\ \partial_t T \\ e_d \end{bmatrix} = \begin{bmatrix} 0 & -d & 0 & 0 & 0 \\ -d & 0 & \frac{ds}{\rho} & \frac{dT}{\rho} & 0 \\ 0 & \frac{-ds}{\rho} & 0 & 0 & \frac{-1}{\rho T} \\ 0 & \frac{-dT}{\rho} & 0 & 0 & \frac{-1}{\rho C_{\mathbf{V}}} \\ 0 & 0 & \frac{1}{\rho T} & \frac{1}{\rho C_{\mathbf{V}}} & 0 \end{bmatrix} \begin{bmatrix} \delta_\rho H \\ \delta_v H \\ \delta_s H \\ \delta_T H \\ dq \end{bmatrix},$$
$$\begin{bmatrix} f_b \\ e_b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_\rho H|_{\partial\mathbf{V}} \\ \delta_v H|_{\partial\mathbf{V}} \end{bmatrix} \tag{22}$$

with external heat flow density q.

Proof. Consider the Hamiltonian given by (14) and its variational derivatives which with the thermodynamic relations $\partial_x u(\rho, s) = \partial_\rho u \partial_x \rho + \partial_s u \partial_x s = \frac{P}{\rho^2} \partial_x \rho + T \partial_x s$ and $C_{\mathbf{V}} = \partial_T u$, yields

$$\delta_\rho H = h + \rho \partial_\rho h = h + \rho^{-1} P, \tag{23}$$
$$\delta_v H = \rho \partial_v h = \rho v, \tag{24}$$
$$\delta_s H = \rho \partial_s h = \rho T, \tag{25}$$
$$\delta_T H = \rho \partial_T h = \rho s + \rho \partial_T u = \rho(s + C_{\mathbf{V}}), \tag{26}$$

where the specific Hamiltonian is expressed by $h = \frac{1}{2} g(\mathbf{v}, \mathbf{v}) + u(\rho, T) + \psi(z) + sT$. From Eq.(22) yields

$$\begin{aligned}
\partial_x \delta_\rho H &= v \partial_x v + \rho^{-1} \partial_x P + T \partial_x s + \partial_x \psi(z), \\
\partial_x \delta_v H &= \rho \partial_x v + v \partial_x \rho, \\
\left(\rho^{-1} \partial_x s\right) \delta_v H &= v \partial_x s, \\
\left(\rho^{-1} \partial_x s\right) \delta_s H &= \partial_s u \partial_x s = T \partial_x s,
\end{aligned}$$

which from Eq.(22) simplifies to (8), (9), (21) and (12) respectively. ■

In order to equate the number of equations to the number of unknown variables, an equation of state $P = P(\rho, s)$ is required. The resulting momentum equation presented in (21) is an approximation of Eq. (9) and is valid only if $\partial P / \partial T = 0$.

Remark 3.1. The application of the dissipativity concepts of Section 2 to the DPHPS of Proposition 3.1 verifies results from DPHPS. In particular the dissipation inequality (5) for this conservative system results in the energy balance

$$\partial_t \mathcal{H} = -\int_A (\rho h + P)\, \mathbf{v} \cdot \mathbf{n}\, dA - \int_{\partial\mathbf{V}} q\, \mathbf{v} \cdot \mathbf{n}\, d\partial\mathbf{V}, \tag{27}$$

where the power transfer through the supply rate functional (15) is equivalent to the power transfer through the ports at the boundary $\partial\mathbf{V}$:

$$\int_{\partial\mathbf{V}} (f_b\, e_b) d\partial\mathbf{V} = \int_A (h + \frac{P}{\rho}) \rho \mathbf{v}\, dA.$$

Furthermore, since direct substitution of (23)-(26) in Eq. (16), yields after some algebra an exact differential, after time integration it yields Eq. (14).

The following result considers the same fluid but expressed in terms of the dynamic variables (ρ, v, u, T) and requires the fact that the used structure matrix corresponds to a Stokes-Dirac structure. This result is not presented here for the sake of space (it is a slight variation of Theorem 3.1).

Proposition 3.2. Assume that a fluid has a storage of energy given by (13). Then the approximated 1-D fluid equations with external (distributed) heat flow (8), (9) and (12) with modified specific internal energy equation given by

$$\rho \partial_t u = v \partial_T P \partial_x T - \partial_x q \tag{28}$$

can be expressed in a port-Hamiltonian form by the Stokes-Dirac structure

$$\begin{bmatrix} \partial_t \rho \\ \partial_t v \\ \partial_t u \\ \partial_t T \\ f_d \end{bmatrix} = \begin{bmatrix} 0 & -d & 0 & 0 & 0 \\ -d & 0 & \Upsilon & \frac{dT}{\rho} & 0 \\ 0 & \Upsilon & 0 & 0 & \frac{-1}{\rho} \\ 0 & \frac{-dT}{\rho} & 0 & 0 & \frac{-1}{\rho C_\forall} \\ 0 & 0 & \frac{1}{\rho} & \frac{1}{\rho C_\forall} & 0 \end{bmatrix} \begin{bmatrix} \delta_\rho H \\ \delta_v H \\ \delta_u H \\ \delta_T H \\ dq \end{bmatrix}$$

$$\begin{bmatrix} f_b \\ e_b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_\rho H|_{\partial\forall} \\ \delta_v H|_{\partial\forall} \end{bmatrix}, \quad \Upsilon = \frac{\partial P}{\partial T} \frac{dT}{\rho^2}. \tag{29}$$

Proof. Consider the Hamiltonian functional defined as (13) with specific hamiltonian $h = \frac{1}{2} g(\mathbf{v}, \mathbf{v}) + u(\rho, T) + \psi(z)$. Since $\partial_x u(\rho, T) = \partial_\rho u \, \partial_x \rho + \partial_T u \, \partial_x T = [\frac{P}{\rho^2} - \frac{T}{\rho^2}(\partial_T P)_\rho] \partial_x \rho + C_\forall \partial_x T$, then

$$\begin{aligned} \delta_\rho \mathcal{H} &= h + \rho \partial_\rho u, \\ \delta_v \mathcal{H} &= \rho v, \\ \delta_u \mathcal{H} &= \rho, \\ \delta_T \mathcal{H} &= \rho \partial_T u = \rho C_\forall, \end{aligned}$$

and from Eq.(29) yields

$$\begin{aligned} \partial_x \delta_v H &= \rho \partial_x v + v \partial_x \rho, \\ \partial_x \delta_\rho H &= v \partial_x v + \frac{1}{\rho} \partial_x P + \\ & C_\forall \partial_x T + \partial_x \psi(z) - \frac{T}{\rho^2} (\partial_T P)_\rho \partial_x \rho, \\ \left(\rho^{-2} \partial_T P \partial_x T\right) \delta_u H &= \rho^{-1} \partial_T P \partial_x T, \\ \left(\rho^{-1} \partial_x T\right) \delta_T H &= \rho \partial_T u = \rho C_\forall, \end{aligned}$$

which can be simplified to obtain Eqs. (8), (9), (28) and (12). ■

In order to equate the number of equations to the number of unknown variables, an equation of state $P = P(\rho, T)$ is required. Equation (28) is only an approximation of (11).

4. CONCLUSIONS

In this paper with the use of a theory of dissipative systems for distributed parameter systems, a theoretical framework for the distributed port-Hamiltonian representation of simple process systems was presented. Although only 1-D models with equilibrium- thermodynamics relations were presented, further generalizations are at present under research.

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